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STRATIFIED THREE PHASE FLOW IN PIPES

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Abstract—The simultaneous flow of water, oil and gas is of practical importance for the oil and gas industry. Many oil and gas pipelines contain these three phases with varying degrees of concentration, depending on the particular situation.

In this work the gas/oil/water holdups for stratified three phase flow are calculated. This information is usually the first step for analyzing the stability of stratified flow and for developing transition criteria.

It is shown that one can obtain three theoretical steady state configurations for stratified flow, but only the configuration with the thinnest total liquid layer is stable and can actually occur.

Taitel & Dukler (*AIChE Jl* **22,** 47–55, 1976) criterion for transition from stratified flow was applied to the three phase flow case and was found to yield good agreement for low gas flow rates.

Key Words: three phase flow, stratified flow, water–oil–gas, flow pattern

INTRODUCTION

Three phase flow of two immiscible liquids and gas is of considerable practical importance for the oil and gas industry. Gas pipelines often contain water and hydrocarbon condensates and oil pipelines can also contain both vapor and water. Water production often increases significantly during the latter stages of a well and use of the conventional approximations of a two phase oil–gas system neglecting the water, or combining the oil and water into a liquid phase, often becomes inaccurate.

In spite of the practical importance, it is surprising how little work has been dedicated to a study of the three phase flow case. Previous publications can be divided into two main categories on the basis of pipe inclination angle. The horizontal and/or slightly inclined case was studied by: Açikgögz *et al.* (1992), Lahey *et al.* (1992), Nuland *et al.* (1991), Lee *et al.* (1993) and Stapelberg *et al.* (1990, 1991 a & b); whereas the vertical case was considered by Chen *et al.* (1990) and Pleshko & Sharma (1990). The work of Stapelberg *et al.* concentrated primarily on slug flow, where the flow is sufficiently chaotic that the liquid phase is essentially a mixture of the two liquids used, water and oil in this case.

Of particular relevance to the present work is the work by Açikgögz *et al.* (1992) and Lee *et al.* (1993). Both reported experimental results for a wide range of flow rates and described in detail the flow patterns observed for the case of horizontal oil–water–air flow. Açikgögz *et al.* (1992) observed a very complex array of flow patterns and described 10 different flow patterns. In their work, the pipe diameter was only 19 mm and stratification was seldom achieved. The Lee *et al.* (1993) experiments were carried out on a test section of 10 cm dia. They observed and classified seven flow patterns, which are similar to the case of two phase flow namely: (1) smooth stratified, (2) wavy stratified, (3) rolling wave, (4) plug flow, (5) slug flow, (6) pseudo slug and (7) annular flow. The first three patterns can be classified as stratified flow and they observed that the oil and the water are generally segregated, with water flowing as a liquid layer at the bottom of the pipe and oil flowing on top. Even for plug flow, the water remains at the bottom, because the agitation of the liquids was not sufficient to mix the oil and the water. On the other hand, in slug flow, pseudo slug and annular flow, oil and water were completely dispersed.

In this work we present a theoretical approach to solve the structure of 3-layer stratified flow. The purpose of the solution is to calculate the levels of the bottom liquid layer and the second layer. These can be used to calculate the holdups of the two liquids and the gas. Once these values are calculated, many other variables that pertain to the two liquids and gas velocities, pressure drop and stability considerations can follow. In particular, the possibility of obtaining multiple solutions is discussed. Also considered is the transition from stratified flow to slug or annular flow.

ANALYSIS

Referring to figure 1, the flow of three fluids is considered: water, oil and gas. It is assumed that the water is heavier than the oil and flows at the bottom. The oil flows in the middle and the gas at the top. A momentum balance for each phase can be written as follows:

$$-A_W\left(\frac{dp}{dx}\right)-\tau_W S_W+\tau_i S_i-\rho_W A_W g\sin\beta=0 \quad [1]$$

$$-A_O\left(\frac{dp}{dx}\right)-\tau_O S_O-\tau_i S_i+\tau_j S_j-\rho_O A_O g\sin\beta=0 \quad [2]$$

$$-A_G\left(\frac{dp}{dx}\right)-\tau_G S_G-\tau_j S_j-\rho_G A_G g\sin\beta=0 \quad [3]$$

where A is a cross sectional area, ρ is density, P is pressure and β is the inclination angle, positive for upward inclination. The subscripts are W for water, O for oil, and G for gas. Five shear stresses are needed to solve [1]–[3]: τ_W, the shear stress acting on the wall wetted by the water S_W; τ_O, the shear stress acting on the wall wetted by the oil S_O; τ_G, the shear stress acting on the wall wetted by the gas S_G; τ_i, the shear stress acting on the oil–water interface S_i; and τ_j, the shear stress acting on the oil–gas interface, S_j. These five shear stresses can be correlated as follows, where U is the average velocity of the fluid in a layer:

$$\tau_W=f_W\frac{\rho_W U_W^2}{2} \quad [4]$$

$$\tau_O=f_O\frac{\rho_O U_O^2}{2} \quad [5]$$

$$\tau_G=f_G\frac{\rho_G U_G^2}{2} \quad [6]$$

$$\tau_i=f_i\frac{\rho_O(U_O-U_W)|U_O-U_W|}{2} \quad [7]$$

$$\tau_j=f_j\frac{\rho_G(U_G-U_O)|U_G-U_O|}{2} \quad [8]$$

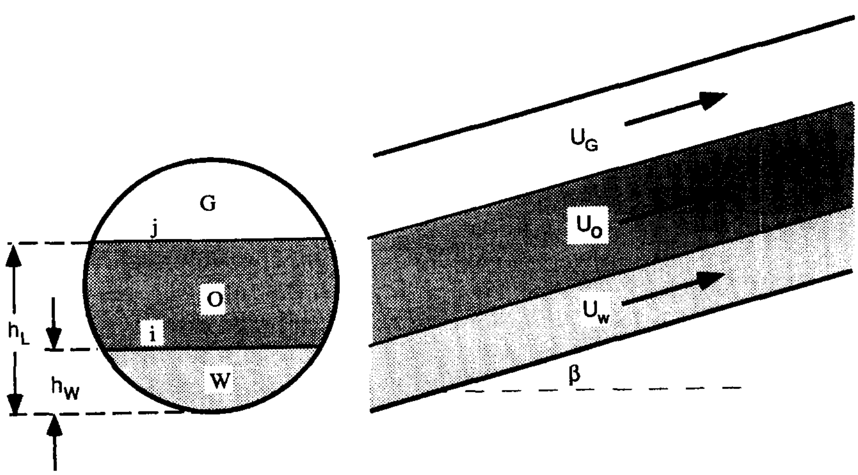

Figure 1. The three layer geometry.

For the shear stresses between the liquids or gas and the pipe surface, the friction factors, f_W, f_O and f_G can be approximated by the correlation

$$f = C\mathrm{R}_e^{-n} \tag{9}$$

where $C = 0.046$, $n = 0.2$ for turbulent flow, and $C = 16$, $n = 1$ for laminar flow. The Reynolds numbers were defined as $\mathrm{R}_{eW} = 4U_W A_W \rho_W / S_W \mu_W$ for the water, $\mathrm{R}_{eO} = 4U_O A_O \rho_O / S_O \mu_O$ for the oil and $\mathrm{R}_{eG} = 4U_G A_G \rho_G / (S_G + S_j)\mu_G$ for the gas. For the interfacial gas–oil shear stress we used a constant value of $f_j = 0.014$, (Cohen & Hanratty 1968) but if the value of f_G was larger than f_j, then $f_j = f_G$ was used. Likewise for the oil–water interface, $f_i = 0.014$ was used or $f_i = f_O$ when the value of f_0 was larger than 0.014.

Summing [1] and [2] yields:

$$-\left(\frac{dp}{dx}\right) - \frac{\tau_L S_L}{A_L} + \frac{\tau_j S_j}{A_L} - \rho_L g \sin\beta = 0 \tag{10}$$

where,

$$\tau_L S_L = \tau_W S_W + \tau_O S_O \tag{11}$$

$$\rho_L = \frac{\rho_W A_W + \rho_O A_O}{A_L} \tag{12}$$

and

$$A_L = A_W + A_O \tag{13}$$

Note that [10] is the combined momentum equation for the liquid phase which is composed of the water and the oil layers. Equations [10] and [3], therefore, have the same form as a two-layer formulation for liquid and gas.

The pressure drop can be eliminated from [3] and [10] to yield:

$$-\frac{\tau_L S_L}{A_L} + \frac{\tau_G S_G}{A_G} + \tau_j S_j\left(\frac{1}{A_L} + \frac{1}{A_G}\right) - (\rho_L - \rho_G) g \sin\beta = 0 \tag{14}$$

In the same way the pressure drop is eliminated from [1] and [2] to yield:

$$-\frac{\tau_W S_W}{A_W} + \frac{\tau_O S_O}{A_O} - \frac{\tau_j S_j}{A_O} + \tau_i S_i\left(\frac{1}{A_W} + \frac{1}{A_O}\right) - (\rho_W - \rho_O) g \sin\beta = 0 \tag{15}$$

Equations [14] and [15] must be solved simultaneously to yield the liquid level h_L, and the water level, h_W. Note that for the case of two layers, one has only a single equation with the exact form as [14]. The case with three layers, however, is much more complicated. Although [14] has the identical format as for the two-layer equation, it must be solved simultaneously with [15] because, unlike the two layer case, τ_L, S_L, ρ_L and A_L, are given in terms of τ_W, τ_O, S_W, S_O, A_W and A_O as can be seen from [11] to [13].

Method of solution

Equations [14] and [15] are two simultaneous equations for the two levels h_W and h_L. The equations are not linear and can have multiple solutions. A method of solution that works quite well is as follows:

1. We start with a guess of h_L at some low value.
2. Once h_L is known, [15] is solved for h_W by a one-dimensional search procedure. For each choice of h_L only one solution for h_W was found.
3. Equation [14] is then tested and if not satisfied, a higher value of h_L is used and step 2 is repeated. Convergence is achieved using the halvation method.
4. Once a solution for h_L and h_W is obtained, a search for additional solutions for the same flow rates is carried out by continuing this procedure with guessed values of h_L larger than the obtained solution.

The advantage of this method is that is covers safely the whole possible steady state solutions that exist and convergence is always assured.

RESULTS AND DISCUSSION

Some examples of the calculations are shown in figures 2–7. Because of the large number of parameters there is no benefit to express the results in a dimensionless form and the results given are for typical examples of water–oil–gas flow to demonstrate the capability of the method. In all cases the pipe diameter is 5 cm, and the properties of water and air are taken at standard room temperature and atmospheric pressure, with the viscosity of water being rounded to 1 cP. Oil density is taken at 800 kg/m^3. We ran two cases for oil viscosities of 1 and 100 cP.

Figures 2–4 show the results for the horizontal case where the liquid level (h_L/D) and the water level (h_W/D) are plotted as a function of the liquid flow rate ($U_{LS} = U_{WS} + U_{OS}$) for equal flow rates of oil and water. Figure 2 is the solution for the two layer case where only water and air are flowing. In figure 3, the oil viscosity is identical to the water viscosity and the liquid level shown in figure 3 is almost identical to the liquid level shown in figure 2. Note that even for the case where we consider the liquid to consist of two layers with identical viscosity and density, the results of the solution for the two layer analysis and the three layer analysis do not have to coincide. That is, the solution for h_L/D for a single water layer is not identical to the solution where the liquid layer is subdivided into two equal viscosity and density layers. Yet the present results indicate that the difference is practically negligible (as we would expect). The bottom water level is shown by a broken line. As can be seen, the water level is usually quite high and the liquid consists mostly of water, especially for low flow rates. This is logical since the oil, being closer to the fast moving gas, is dragged by the gas to higher velocities than the water layer at the bottom. Because the oil travels faster than the water, the *in situ* holdup of oil is lower than that of water.

When oil viscosity is increased to 100 cP, figure 4 shows that the total liquid level increases, especially for low liquid rates, and the relative level of the water decreases (although not dramatically). This is also expected since the more viscous oil will move slower. As a result, its relative holdup will increase as well as the total holdup of the liquid (water + oil). The phenomenon is enhanced for low liquid flow rates because the effect of viscosity is lower for high, turbulent flow rates compared to low, laminar flow rates.

Note that, as for the case of two layer systems, a single solution exists for the horizontal case, for specific liquids and gas flow rates. Thus, for a given set of liquid properties, pipe diameter and water, oil and gas flow rates, there is a single solution for the thickness of the water and the oil layers.

Figures 5 to 7 show that more than one solution is possible for the upward inclined case. Figure 5 is the solution for the two layer case for water–air. The possibility of obtaining three solutions for the case of two layers gas–liquid flow has been observed and discussed by others (Baker & Gravestock 1987; Landman 1991 a & b; Barnea & Taitel 1992). Figures 6 and 7 show that also for the case of three phase flow, three solutions are possible for the low liquid flow rates. In this region of low liquid flow rates the relative oil layer is very thin.

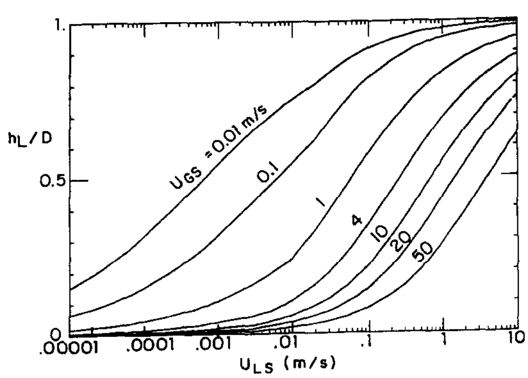

Figure 2. Liquid levels for water–air flow in horizontal pipes.

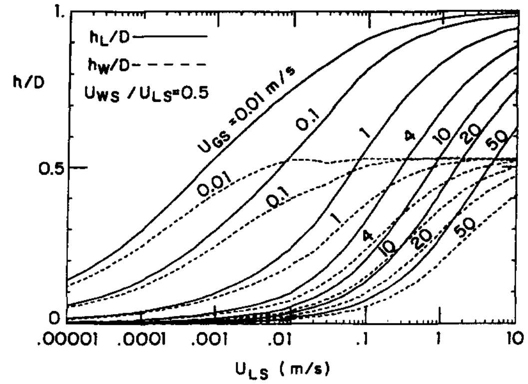

Figure 3. Liquid levels for water–oil–air flow in horizontal pipes. Oil viscosity 1 cP. Water flow rate ratio 50%.

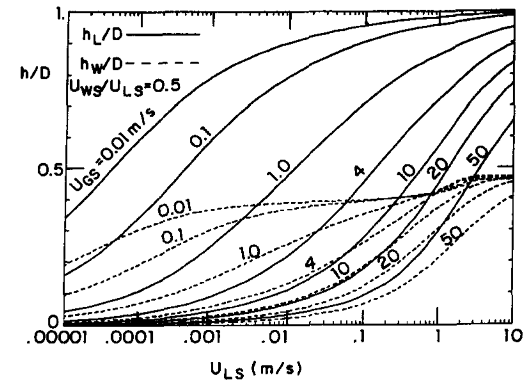

Figure 4. Liquid levels for water–oil–air flow in horizontal pipes. Oil viscosity 100 cP. Water flow rate ratio 50%.

One would expect that the oil layer will be thinner than the water layer (for equal flow rates) because the oil layer, in between the water and the gas layer will have a velocity which is between the water and the fast moving gas. However, the extremely thin oil layer in the region of low U_{LS} is by no means intuitive. Another, non-intuitive observation is that, unlike the horizontal case where the increase of viscosity causes a significant increase in the liquid level, for the case of upward flow (see figure 7) the viscosity has little effect on the liquid level. Only for gas velocity in the range of 8–20 m/s there is a noticeable change of liquid level due to an increase in the oil viscosity.

The multiple solutions

When more than a single steady state solution exists, it is important to identify the physically realistic solutions and whether it is possible to have more than one solution in practice. This problem for the case of two layers was thoroughly discussed by Barnea & Taitel (1992) who analyzed in detail the stability of the steady state solutions obtained for the case of two layers. It was concluded that, when three steady state solutions exist, the only realistic solution is the solution with the most thin liquid level. The other two "thick" solutions were found to be unstable. Likewise we assume here that also for the three layers the only valid physical solution is the first (thin) solution.

Flow pattern transition

Transition from stratified flow (to slug or annular flow) is believed to be the result of Kelvin–Helmholtz instability that causes waves on the gas–liquid interface to be unstable and grow up to the point that they touch the top part of the pipe, resulting in transition to slug or annular flow (Taitel & Dukler 1976; Lin & Hanratty 1986: Wu *et al.* 1987; Barnea 1987; Taitel 1990; Barnea 1991; Barnea & Taitel 1992). In this work we used the method proposed by Taitel & Dukler (1976)

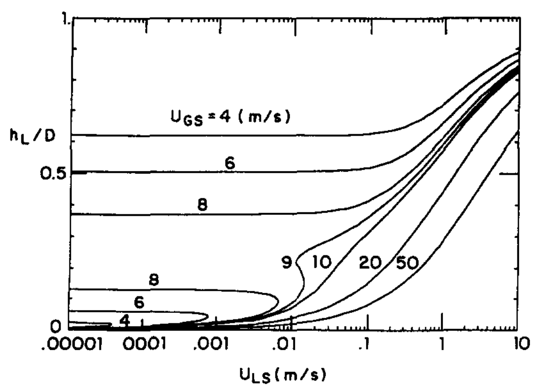

Figure 5. Liquid levels for water–air flow in pipe inclined upward 1°.

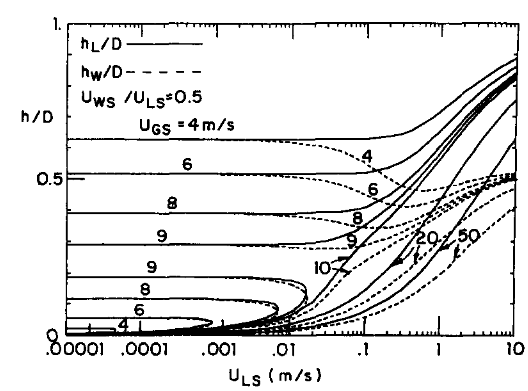

Figure 6. Liquid levels for water–oil–air flow in pipe inclined upward 1°. Oil viscosity 1 cP. Water flow rate ratio 50%.

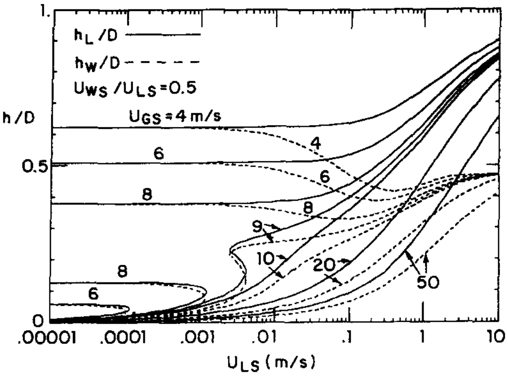

Figure 7. Liquid levels for water–oil–air flow in pipe inclined upward 1°. Oil viscosity 100 cP. Water flow rate ratio 50%.

to calculate the location of the transition boundaries. The extension of the more advanced methods such as those proposed by Lin & Hanratty (1986), Wu (1987) and Barnea (1991) to the three phase geometry is not trivial. Barnea (1991) showed that the Taitel & Dukler (1976) method is, in fact, quite accurate for viscosities up to 100 cP. Thus, in this work the Taitel & Dukler (1976) transition from stratified flow is considered, namely the liquid level is unstable when

$$U_G - U_O > \left(1 - \frac{h_L}{D}\right)\sqrt{\frac{(\rho_O - \rho_G)gA_G\cos\beta}{\rho_G S_i}} \qquad [16]$$

and once [16] is satisfied, slug flow will exist for high liquid holdup and annular flow for low liquid holdup ($h_L/D = 0.35$, Barnea *et al.* 1982).

Figure 8 shows results for the horizontal case for water flow rate fractions of 1.0, 0.5, 0.1 and 0. The solid line represents the case for a water fraction of 1.0 when the liquid phase is all water (the two layer model).

Introducing an oil layer of higher viscosity reduces the local velocities and increases the liquid level in the pipe. Since the transition from stratified flow is directly related to the liquid level in the pipe (for a given gas flow rate), then when the water flow rate fraction decreases, the liquid level increases and the transition from stratified flow occurs at lower liquid and gas flow rates.

A similar effect can be seen in figure 9 for the inclined case. As the oil flow rate fraction is increased, the region of stratified flow decreases. However, this statement needs some elaboration. Previously, the observation was made that, unlike the horizontal case where the liquid level changes considerably when increasing the oil fraction, this is not the case for the inclined case where the liquid level usually is little affected by the increase of oil fraction. However, as mentioned

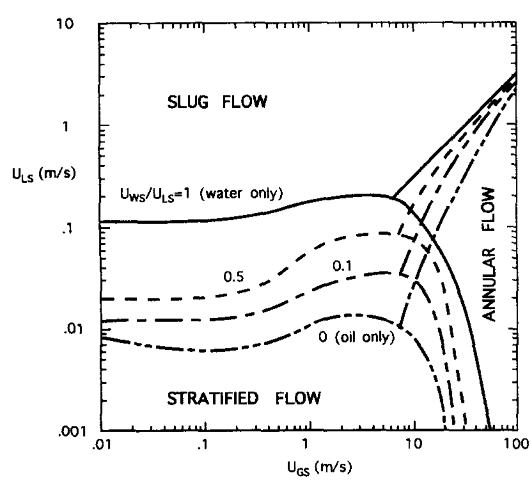

Figure 8. Flow patterns for water–oil–air flow in horizontal pipes. Oil viscosity 100 cP.

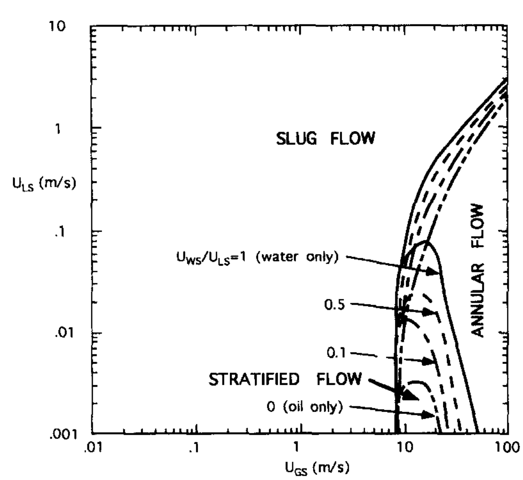

Figure 9. Flow patterns for water–oil–air flow in pipe inclined upward 1°. Oil viscosity 100 cP.

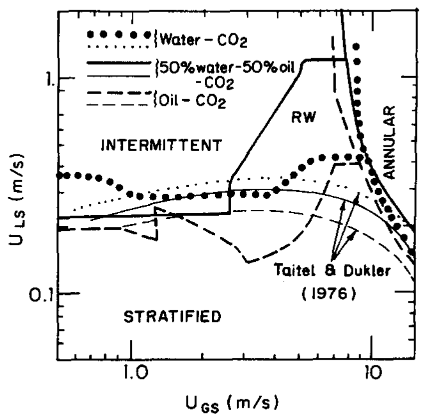

Figure 10. Flow pattern transition—comparison with experiment (Lee *et al.* 1993). Oil viscosity 15 cP.

previously, there are considerable differences in h_L between the two phase and the three phase cases in the range of gas flow rate around 8–20 m/s (figures 5 and 7). This is exactly the range in which transition from stratified flow occurs. Therefore, increasing the oil flow rate fraction has a substantial influence on this transition boundary.

Figure 10 shows some experimental results for the flow patterns taken from Lee *et al.* (1993) for water–oil–CO_2 flow in 10 cm dia horizontal pipe. The oil viscosity is 15 cP. The figure contains the transition boundaries for water–CO_2, oil–CO_2 and water–oil–CO_2 for which the flow rates of water and oil are the same. They identified three basic flow patterns: stratified flow, intermittent flow and annular flow. However, they subdivided the stratified flow region into three subregions: stratified smooth (SS), wavy stratified (WS) and rolling waves (RW). Likewise the intermittent region was subdivided into plug flow (P), slug flow (S) and pseudo slug (PS) flow. For the sake of clarity figure 10 contains only the major flow patterns, that is, stratified flow, intermittent flow and annular flow.

The figure shows that for low gas flow rates the transition to intermittent flow "moves" downward as oil fraction is increased. For the case of water–oil–CO_2 at high gas flow rate the region of stratified flow (the subregion of rolling waves) is considerably expanded compared to this region for the two phase case water–CO_2 or oil–CO_2. Figure 10 shows that it is not easy to find a logical consistent trend as one changes from pure water to pure oil and the apparent increase of the roll wave region for the water–oil–CO_2 system remains unexplained.

The model of Taitel & Dukler (1976) shows relatively good agreement for the stratified–intermittent transition for low gas flow rates. For high flow rates the agreement with experiments is not very good. Obviously the Taitel & Dukler model is unable to explain the unexpected considerable increase of the region of roll waves for the three phase case (this phenomenon occurs also at 75% water–25% oil and 25% water–75% oil as reported by Lee *et al.*). However, more work and additional data are needed to clarify accurately this special phenomenon of the enlargement of the roll wave region due to the presence of both oil and water within the liquid phase.

SUMMARY AND CONCLUSIONS

Three phase (two liquids and gas) stratified flow in a pipe is considered. The levels of the two liquids in the pipe for any given set of flow rates is calculated. The results are presented on h/D (h_L and h_W) vs $U_{LS}(=U_{WS}+U_{OS})$ maps for parametric values of U_{GS}. Examples are given for the case of water–oil–air flowing in horizontal and upwardly inclined pipes of 5 cm dia.

It is shown that, like the two phase case, three steady state solutions can be obtained for the upward inclined case. By analogy to the two phase flow case, it is assumed that the most thin layer solution is the one that can physically occur.

Information regarding the liquid and oil levels is in fact an essential step for the calculation of holdup, pressure drop and the development of flow pattern transition criteria. The Taitel & Dukler (1976) criterion for the transition from stratified flow is found to agree with some experimental data at relatively low gas flow rates.

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